A Semiempirical Solution for Local Heat Transfer Coefficients for Flow in Nonparallel Passageways

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A method, based upon the pertinent flat-plate heat transfer equation, is presented for computing the local heat transfer coefficients for a boundary layer subjected to streamwise velocity and pressure gradients. No extensive mathematical background is required as the complexity of a rigorous solution for this type of problem is avoided. The validity of the method for gases is demonstrated by comparison of the predicted coefficients with the experimental data for two widely different problems.

At the present time no good solution is available to the engineer for predicting the local heat transfer coefficients in a passageway with accelerated or decelerated flow. The empirical Colburn equation fails both qualitatively and quantitatively in many problems. Exact solutions must satisfy a number of fundamental relationships: the integral momentum equation, the energy equation, and the continuity equation. Too, assumptions must be made on the velocity and temperature profiles as well as approximations on skin friction and the relationship between heat and momentum transfer. Because of these complexities, it is doubtful whether an exact solution is feasible for many problems where cost or time is a factor.

It is the object of this paper to present a simple yet reliable method of analysis that can be readily handled by the engineer for a wide variety of applications. The method is based on the appropriate flat-plate equation for local heat transfer coefficients without pressure gradient—appropriate in the sense that the type of flow is similar to that of the analysis.

METHOD

1. Select from the literature a flat-plate equation which is known to be accurate for turbulent (or laminar) flow and for the fluid under consideration.

2. Extend the equation to include variable velocity by differentiation:

$$dh = \frac{\partial h}{\partial x}\,dx + \frac{\partial h}{\partial U}\,dU + \frac{\partial h}{\partial \rho}\,d\rho$$

For the turbulent-flow examples of this paper the flat-plate equation of Eckert (*1*) was selected:

$$St = \frac{Nu}{Re\,Pr} = \frac{0.0296/Re^{0.2}}{1 + [0.0296/Re^{0.2}]^{1/2}\left[5(Pr - 1) + 5\ln\frac{5Pr + 1}{6}\right]}$$

This equation can be arranged into the form

$$h = \frac{QU\rho}{Re^{0.2} + N\,Re^{0.1}}$$

The partial derivatives of h are as follows:

$$\frac{\partial h}{\partial x} = -\frac{Q\rho U}{x}\left[\frac{0.2\,Re^{0.2} + 0.1N\,Re^{0.1}}{[Re^{0.2} + N\,Re^{0.1}]^2}\right]$$

$$\frac{\partial h}{\partial U} = Q\rho\left[\frac{0.8\,Re^{0.2} + 0.9N\,Re^{0.1}}{[Re^{0.2} + N\,Re^{0.1}]^2}\right]$$

$$\frac{\partial h}{\partial \rho} = QU\left[\frac{0.8\,Re^{0.2} + 0.9N\,Re^{0.1}}{[Re^{0.2} + N\,Re^{0.1}]^2}\right]$$

3. When the leading-edge coefficient is infinite (the usual case), utilize the flat-plate equation to obtain an initial coefficient near the leading edge. The choice of this initial increment is arbitrary but should be as small as practical, say, 1/100 in.

4. Select increments throughout the passageway (10 steps are usually sufficient) and find h_2, etc.:

$$h_2 = h_1 + \frac{\partial h}{\partial x}\Delta x_{12} + \frac{\partial h}{\partial U}\Delta U_{12} + \frac{\partial h}{\partial \rho}\Delta \rho_{12}$$

(For the examples to follow all properties were evaluated at the mean local film temperature and flow conditions assumed to be isentropic.)

Note that the qualitative change in h can in part be rationalized by inspection of dh:

$$dh = \frac{\partial h}{\partial x}\,dx + \frac{\partial h}{\partial U}\,dU + \frac{\partial h}{\partial \rho}\,d\rho$$

In the convergent section of a nozzle the first two terms are significant, with the velocity term being predominant except near the leading edge, where the first term is of prime importance. The third term is relatively small until the region near the throat is reached, but then large negative values of this term cause the maximum coefficient to be realized immediately before the throat. After the throat the density term is the controlling factor and the coefficient rapidly decreases in value. These effects, of course, are further influenced by the pressure gradient, and this influence is neglected in this analysis.

COMPARISON WITH EXPERIMENTAL DATA

The proposed method was used to predict coefficients for two widely different flow problems recently investigated

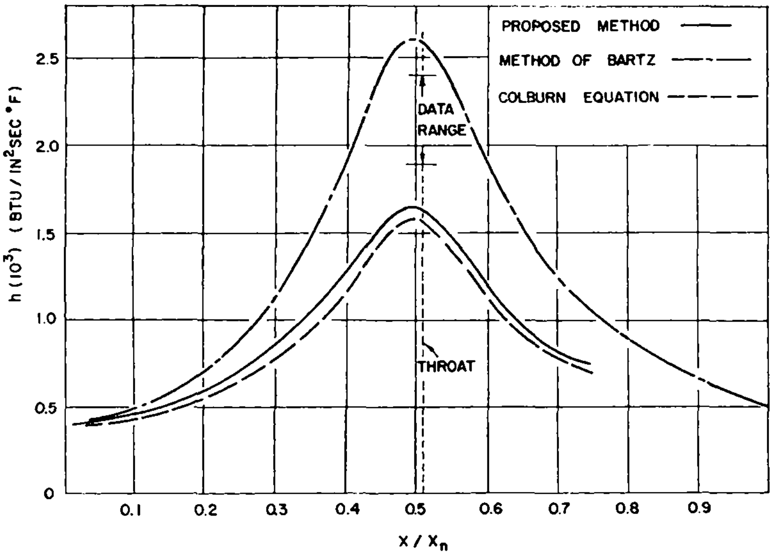

Fig. 1. Local heat transfer coefficients h for a nozzle with accelerated velocities (outlet velocity at mach 2).

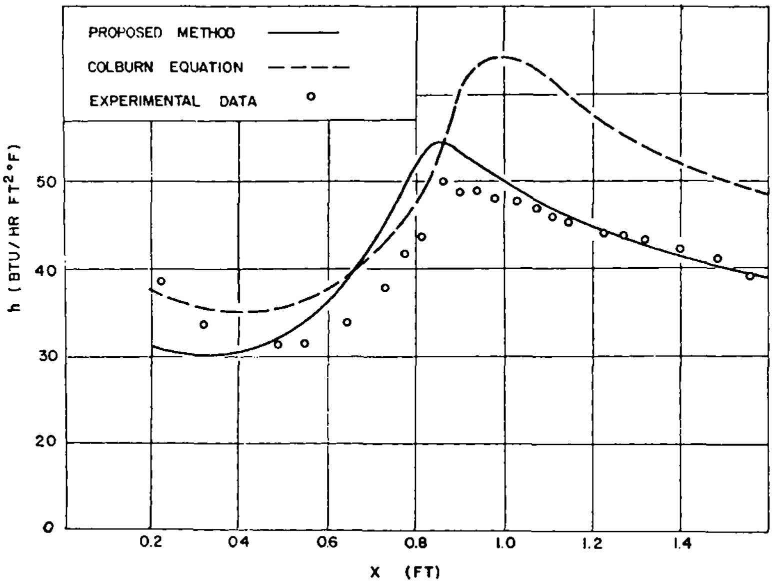

Fig. 2. Local heat transfer coefficients h for a flat plate with accelerated and decelerated subsonic velocities.

by Bartz (*2*) and by Seban and Doughty (*3*). In the Bartz problem the flow was accelerated to supersonic velocities with the pressure continually decreasing in the direction of flow (negative pressure gradient); in the Seban-Doughty problem the flow was accelerated to a subsonic velocity (negative pressure gradient) and then decelerated (positive pressure gradient).

Bartz made a rigorous, but complex, analytical solution for the flow of combustion gases at high temperature (4,000 °F.) through a convergent-divergent nozzle. Although not included in the paper, experimental data (*4*) were obtained for a nozzle which was the prototype of the analytical solution. The range of values of these data at the throat of the nozzle is shown in Figure 1, along with Bartz's predictions (his Case IIa) and those made by the relatively simple analysis of this paper. The proposed method and the Colburn equation yield results on the low side, while the results of Bartz's method* are on the high side of the experimental data. It would appear for this particular problem that the deviations of the two methods from the correct values are probably quite comparable.

Seban and Doughty experimentally determined local heat transfer coefficients over the surface of a flat plate which was located in the converging-diverging section of a wind tunnel. The experimental and analytical results for their run 99 are shown in Figure 2 along with values calculated by the methods of this paper (*5*). By the proposed method both qualitative and quantitative correlations are obtained with the experimental data.

The Colburn coefficients calculated by Seban Doughty (Figure 2) correlate reasonably well with their experimental data in the accelerated-flow region. The Colburn coefficients are in neither qualitative nor quantitative agreement with the experimental data over the whole flat plate.

It should be mentioned that the close quantitative correlation of the proposed method with the Seban-Doughty data is somewhat surprising. Qualitative agreement was expected from the differentiation procedure and step-by-step solution. But the flat-plate equations should not reflect strong pressure gradients as the data are for zero gradients. Thus the proposed method should predict values on the low side in the regions of accelerated flow and values on the high side in regions of decelerated flow. It is believed, however, that the results will be superior in most cases to those from the Colburn equation.

*Bartz has recently made (private communication) a second calculation which reduces his throat value to 2.35 (10^{-3}).

NOTATION

c_p = heat capacity at constant pressure
h = local heat transfer coefficient

$$N = [0.0296]^{1/2}\left[5(Pr - 1) + 5 \ln \frac{5Pr + 1}{6}\right]$$

Nu = Nusselt number
Pr = Prandtl number
Q = 0.0296 c_p
Re = Reynolds number $\frac{\rho U_x}{\mu}$
St = Stanton number
U = local-main-stream velocity
x = distance from the leading edge
x_n = nozzle length

Greek Letters

ρ = density
μ = viscosity

LITERATURE CITED

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